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Explicit Solution of Optimal Power Flow in Power System Including Power losses and Generation Limits

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Abstract: *Minimizing the generation cost is one of the main concerns in power system and it is usually defined as the Optimal Power Flow (OPF) problem, the equality constraint of this optimization problem is nonlinear due to the power losses of the transmission lines, where the power losses can be given either by a nonlinear function of generation powers or it can determined from the load flow, which is also a set of non linear equations, due to the nonlinearity of the power losses all the optimization techniques either deterministic or heuristic are locating the solution of OPF iteratively, in this paper an explicit solution of the OPF without need of iteration is proposed, the proposed equation depends on the B coefficient matrices, combined with the Lagrange method to cover the power losses and the generation limit constraints. The proposed equation was used to determine the OPF of IEEE 26 buses system, the results show that the proposed equation gave the same minimum cost that can be obtained from the Lagrange method.*

Keywords - *Optimal power flow, Lagrange, Power losses, Explicit solution, Generation limits.*

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I. Introduction

In Optimal Power Flow (OPF) of power systems, the power losses in transmission line is calculated based on the load flow program[1]-[4] or by using B coefficient formula[5]-[9], in both cases, the power losses is described by a nonlinear equation, so the OPF problem is solved using iterative optimization techniques, where a lot of heuristic optimization algorithms [1], [3]-[5] and deterministic optimization methods[10]-[11] were used to solve the OPF problem. The generation limits are also must be included in the OPF, when any of generation power violates its limits, then the value of it is kept on the limit that it violates.

As it is mentioned earlier that the power losses can be calculated by B coefficient matrices, where the power losses in this case is a direct function of the generation powers, this way has its advantage, but at the same time these matrices give an accurate power losses near the operating values that the B matrices are calculated. In the proposed method, this will be not problem, since these matrices will be determined at the values of the optimal generation powers that are obtained from Lagrange method without including losses, since the power losses is very small compared of the total demand power, so the generation power including losses will be near the values that obtained when the power losses are not included.

After calculating the B coefficient matrices , the power losses equation is modified to be given in terms of the Lagrange multiplier λ , where the function becomes quadratic equation in terms of λ, and the value of λ at the optimal solution λmax can be determined by solving a quadratic equation, where the solution is given explicitly. Including the generation limits will not change the degree of the power losses equation but it affects the coefficient of the quadratic equation, the coefficient of power losses equation in term of λ including generation limits is derived in this paper. The OPF of IEEE 26 buses system is calculated based on the proposed equation and the Lagrange method for several values of loads, the results show that the minimum generation cost is the same in both cases, in some of load cases the generation limits are violated and the results are also show that both methods have the same minimum generation cost.

II. Power losses' Modeling

The B coefficient matrices are used to calculate the power losses of the transmission lines in terms of the generation power, where the power losses are given as follows:

$$\boldsymbol{P_{loss} = P_g' B P_g + B_0 P_g + B_{00}} \tag{1}$$

Where,

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B_{11}} & \boldsymbol{B_{12}} & \cdots & \boldsymbol{B_{1ng}} \\ \boldsymbol{B_{21}} & \boldsymbol{B_{22}} & \cdots & \boldsymbol{B_{2ng}} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{B_{ng1}} & \boldsymbol{B_{ng2}} & \cdots & \boldsymbol{B_{ngng}} \end{bmatrix}_{ng\times ng}, \quad \boldsymbol{B_0} = \lfloor \boldsymbol{B_{01}} \quad \boldsymbol{B_{02}} \quad \cdots \quad \boldsymbol{B_{0ng}} \rfloor_{1\times ng}.$$

The OPF can be obtained by minimizing the total cost of the generation units, where the generation cost is usually represented by quadratic equation of the generation power, for each generation plant the cost can be given as follows:

$$\mathbf{cost}_i = \gamma_i P_{gi}^2 + \beta_i P_{gi} + \alpha_i \qquad (2)$$

When the power losses are not included in the OPF problem, there is an exact solution of the optimal generation power in terms of the Lagrange multiplier λ as follows:

$$P_{gi} = \frac{\lambda - \beta_i}{2\gamma_i} \qquad (3)$$

$$\lambda = \frac{P_D + \sum_{i=1}^{ng} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{ng} \frac{1}{2\gamma_i}} \qquad (4)$$

Where, P_D is the total demand Power.

Now, if the power losses formula is written in terms of λ by substitute equ.(3) in equ.(1)

$$P_{loss} = \begin{bmatrix} \frac{\lambda-\beta_1}{2\gamma_1} & \frac{\lambda-\beta_2}{2\gamma_2} & \cdots & \frac{\lambda-\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} B \begin{bmatrix} \frac{\lambda-\beta_1}{2\gamma_1} \\ \frac{\lambda-\beta_2}{2\gamma_2} \\ \vdots \\ \frac{\lambda-\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} + B_0 \begin{bmatrix} \frac{\lambda-\beta_1}{2\gamma_1} \\ \frac{\lambda-\beta_2}{2\gamma_2} \\ \vdots \\ \frac{\lambda-\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} + B_{00} \qquad (5)$$

$$\begin{aligned} P_{loss} = {} & \lambda^2 \begin{bmatrix} \frac{1}{2\gamma_1} & \frac{1}{2\gamma_2} & \cdots & \frac{1}{2\gamma_{ng}} \end{bmatrix} B \begin{bmatrix} \frac{1}{2\gamma_1} \\ \frac{1}{2\gamma_2} \\ \vdots \\ \frac{1}{2\gamma_{ng}} \end{bmatrix} - \lambda \begin{bmatrix} \frac{\beta_1}{2\gamma_1} & \frac{\beta_2}{2\gamma_2} & \cdots & \frac{\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} B \begin{bmatrix} \frac{1}{2\gamma_1} \\ \frac{1}{2\gamma_2} \\ \vdots \\ \frac{1}{2\gamma_{ng}} \end{bmatrix} \\ & - \lambda \begin{bmatrix} \frac{1}{2\gamma_1} & \frac{1}{2\gamma_2} & \cdots & \frac{1}{2\gamma_{ng}} \end{bmatrix} B \begin{bmatrix} \frac{\beta_1}{2\gamma_1} \\ \frac{\beta_2}{2\gamma_2} \\ \vdots \\ \frac{\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} + \lambda B_0 \begin{bmatrix} \frac{1}{2\gamma_1} \\ \frac{1}{2\gamma_2} \\ \vdots \\ \frac{1}{2\gamma_{ng}} \end{bmatrix} - B_0 \begin{bmatrix} \frac{\beta_1}{2\gamma_1} \\ \frac{\beta_2}{2\gamma_2} \\ \vdots \\ \frac{\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} + B_{00} \end{aligned} \qquad (6)$$

$$P_{loss} = a\lambda^2 + b\lambda + c \qquad (7)$$

Where

$$a = \begin{bmatrix} \frac{1}{2\gamma_1} & \frac{1}{2\gamma_2} & \cdots & \frac{1}{2\gamma_{ng}} \end{bmatrix}_{ng\times 1} B \begin{bmatrix} \frac{1}{2\gamma_1} \\ \frac{1}{2\gamma_2} \\ \vdots \\ \frac{1}{2\gamma_{ng}} \end{bmatrix}_{1\times ng}$$

$$b = -\begin{bmatrix} \frac{\beta_1}{2\gamma_1} & \frac{\beta_2}{2\gamma_2} & \cdots & \frac{\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} B \begin{bmatrix} \frac{1}{2\gamma_1} \\ \frac{1}{2\gamma_2} \\ \vdots \\ \frac{1}{2\gamma_{ng}} \end{bmatrix} - \begin{bmatrix} \frac{1}{2\gamma_1} & \frac{1}{2\gamma_2} & \cdots & \frac{1}{2\gamma_{ng}} \end{bmatrix} B \begin{bmatrix} \frac{\beta_1}{2\gamma_1} \\ \frac{\beta_2}{2\gamma_2} \\ \vdots \\ \frac{\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} + B_0 \begin{bmatrix} \frac{1}{2\gamma_1} \\ \frac{1}{2\gamma_2} \\ \vdots \\ \frac{1}{2\gamma_{ng}} \end{bmatrix}$$

$$c = \begin{bmatrix} \frac{\beta_1}{2\gamma_1} & \frac{\beta_2}{2\gamma_2} & \cdots & \frac{\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} B \begin{bmatrix} \frac{\beta_1}{2\gamma_1} \\ \frac{\beta_2}{2\gamma_2} \\ \vdots \\ \frac{\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} - B_0 \begin{bmatrix} \frac{\beta_1}{2\gamma_1} \\ \frac{\beta_2}{2\gamma_2} \\ \vdots \\ \frac{\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} + B_{00}$$

Now, if the power losses is included in OPF equations, the solution of the optimization problem will be as follows:

$$P_D + a\lambda^2 + b\lambda + c - \sum_{i=1}^{ng} P_{gi} = 0 \tag{8}$$

$$\sum_{I=1}^{ng} \frac{\lambda - \beta_i}{2\gamma_i} = P_D + a\lambda^2 + b\lambda + c \tag{9}$$

$$a\lambda^2 + (b - \sum_{i=1}^{ng} \frac{1}{2\gamma_i})\lambda + c + \sum_{i=1}^{ng} \frac{\beta_i}{2\gamma_i} = 0 \tag{10}$$

$$a\lambda^2 + b'\lambda + c' = 0 \tag{11}$$

Where: $b' = b - \sum_{i=1}^{ng} \frac{1}{2\gamma_i}$, and $c' = c + \sum_{i=1}^{ng} \frac{\beta_i}{2\gamma_i}$

The solution of equ.(10) is

$$\lambda = \frac{-b' \pm \sqrt{b'^2 - 4ac'}}{2a} \tag{12}$$

Then, optimal generation power can be calculated from equ.(3), and power losses from equ.(7)

III. Including Generation Limits

If one of the optimal generation power violates its limits, this generation power is kept on its limit, and the other optimal generation powers are calculated again, let for example P_{gm} is reached its maximum or its minimum limits, then the power losses can be evaluate as follows:

Starting from equ.(5), and evaluate og the generation power that is violate its limits, by its value, then the equation will be :

$$P_{loss} = \begin{bmatrix} \frac{\lambda - \beta_1}{2\gamma_1} & \frac{\lambda - \beta_2}{2\gamma_2} & \cdots & P_{gm} & \frac{\lambda - \beta_{gm+1}}{2\gamma_{gm+1}} & \cdots & \frac{\lambda - \beta_{ng}}{2\gamma_{ng}} \end{bmatrix} B \begin{bmatrix} \frac{\lambda - \beta_1}{2\gamma_1} \\ \frac{\lambda - \beta_2}{2\gamma_2} \\ \vdots \\ P_{gm} \\ \frac{\lambda - \beta_{gm+1}}{2\gamma_{gm+1}} \\ \vdots \\ \frac{\lambda - \beta_{ng}}{2\gamma_{ng}} \end{bmatrix} + B_0 \begin{bmatrix} \frac{\lambda - \beta_1}{2\gamma_1} \\ \frac{\lambda - \beta_2}{2\gamma_2} \\ \vdots \\ P_{gm} \\ \frac{\lambda - \beta_{gm+1}}{2\gamma_{gm+1}} \\ \vdots \\ \frac{\lambda - \beta_{ng}}{2\gamma_{ng}} \end{bmatrix} + B_{00} \tag{13}$$

Based on equ.(13), the losses power still can be represented by quadratic equations but with different coefficient as follows:

$$p_{loss} = a_{\text{mod}}\lambda^2 + b_{\text{mod}}\lambda + c_{\text{mod}} \tag{14}$$

Where:

$$a_{\text{mod}} = \begin{bmatrix} \frac{1}{2\gamma_1} & \frac{1}{2\gamma_2} & \cdots & \frac{1}{2\gamma_{gm-1}} & \frac{1}{2\gamma_{gm+1}} & \cdots & \frac{1}{2\gamma_{ng}} \end{bmatrix} B' \begin{bmatrix} \frac{1}{2\gamma_1} \\ \frac{1}{2\gamma_2} \\ \vdots \\ \frac{1}{2\gamma_{gm-1}} \\ \frac{1}{2\gamma_{gm+1}} \\ \vdots \\ \frac{1}{2\gamma_{ng}} \end{bmatrix} \tag{15}$$

$$B' = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1,m-1} & B_{1,gm+1} & \cdots & B_{1ng} \\ B_{21} & B_{22} & \cdots & B_{2,gm-1} & B_{2,gm+1} & \cdots & B_{2ng} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ B_{gm-1,1} & B_{gm-1,2} & \cdots & B_{gm-1,gm-1} & B_{gm-1,gm+1} & \cdots & B_{gm-1,ng} \\ B_{gm+1,1} & B_{gm+1,22} & \cdots & B_{gm+1,gm-1} & B_{(gm+1,gm+1} & \cdots & B_{(gm+1,ng} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ B_{ng1} & B_{ng2} & \cdots & B_{ng(gm-1)} & B_{ng(gm+1)} & \cdots & B_{ngng} \end{bmatrix}, \quad B'_0 = \begin{bmatrix} B_{01} \\ B_{02} \\ \vdots \\ B_{0,gm-1} \\ B_{0,gm+1} \\ \vdots \\ B_{0,ng} \end{bmatrix} \tag{16}$$

$$b_{\text{mod}} = 2\begin{bmatrix} \frac{-\beta_1}{2\gamma_1} & \frac{-\beta_2}{2\gamma_2} & \cdots & \frac{-\beta_{gm-1}}{2\gamma_{gm-1}} & \frac{-\beta_{gm+1}}{2\gamma_{gm+1}} & \cdots & \frac{-\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} B' \begin{bmatrix} \frac{1}{2\gamma_1} \\ \frac{1}{2\gamma_2} \\ \vdots \\ \frac{1}{2\gamma_{gm-1}} \\ \frac{1}{2\gamma_{gm+1}} \\ \vdots \\ \frac{1}{2\gamma_{ng}} \end{bmatrix} + 2\text{P}_{\text{gm}} \begin{bmatrix} \frac{1}{2\gamma_1} & \frac{1}{2\gamma_2} & \cdots & \frac{1}{2\gamma_{gm-1}} & \frac{1}{2\gamma_{gm+1}} & \cdots & \frac{1}{2\gamma_{ng}} \end{bmatrix} \begin{bmatrix} \text{B}_{\text{1gm}} \\ \text{B}_{\text{2gm}} \\ \vdots \\ \text{B}_{\text{(gm-g1)m}} \\ \text{B}_{\text{(gm+1)gm}} \\ \vdots \\ \text{B}_{\text{nggm}} \end{bmatrix} + B'_0 \begin{bmatrix} \frac{1}{2\gamma_1} \\ \frac{1}{2\gamma_2} \\ \vdots \\ \frac{1}{2\gamma_{gm-1}} \\ \frac{1}{2\gamma_{gm+1}} \\ \vdots \\ \frac{1}{2\gamma_{ng}} \end{bmatrix} \tag{17}$$

$$c_{\text{mod}} = \begin{bmatrix} \frac{-\beta_1}{2\gamma_1} & \frac{-\beta_2}{2\gamma_2} & \cdots & \frac{-\beta_{gm-1}}{2\gamma_{gm-1}} & \frac{-\beta_{gm+1}}{2\gamma_{gm+1}} & \cdots & \frac{-\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} B' \begin{bmatrix} \frac{-\beta_1}{2\gamma_1}, \\ \frac{-\beta_2}{2\gamma_2} \\ \vdots \\ \frac{-\beta_{gm-1}}{2\gamma_{gm-1}} \\ \frac{-\beta_{gm+1}}{2\gamma_{gm+1}} \\ \vdots \\ \frac{-\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} + B'_o \begin{bmatrix} \frac{-\beta_1}{2\gamma_1} \\ \frac{-\beta_2}{2\gamma_2} \\ \vdots \\ \frac{-\beta_{gm-1}}{2\gamma_{gm-1}} \\ \frac{-\beta_{gm+1}}{2\gamma_{gm+1}} \\ \vdots \\ \frac{-\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} + 2\begin{bmatrix} \frac{-\beta_1}{2\gamma_1}, \\ \frac{-\beta_2}{2\gamma_2} \\ \vdots \\ \frac{-\beta_{gm-1}}{2\gamma_{gm-1}} \\ \frac{-\beta_{gm+1}}{2\gamma_{gm+1}} \\ \vdots \\ \frac{-\beta_{ng}}{2\gamma_{ng}} \end{bmatrix} \begin{bmatrix} B_{gm,1} \\ B_{gm,2} \\ \vdots \\ B_{gm,gm-1} \\ B_{gm,gm+1} \\ \vdots \\ B_{gm,ng} \end{bmatrix} p_{gm} + B_{00} + P_{gm}^T B_{gmgm} P_{gm} + B_{0gm} P_{gm} \tag{18}$$

IV. Results and Discussion

Now, the OPF of IEEE 26 buses power system, will be determined using the proposed formula, then the result will be compared to that obtained from the Lagrange multiplier method, the B coefficient model here in the Lagrange method is calculated at each iteration until the optimal power is achieved, while in the proposed formula, the B coefficient is calculated once, the results are shown in Fig.1, Fig.2 and Fig.3, where the load is changed 10 times (all the load buses' power is scaled at the same ratio), B matrices must evaluated for each load, the generation cost using the two methods is the same, the total power losses using Lagrange multiplier is slightly less than the proposed method as it is shown in Fig.3, while Fig.2 shows the generation power from each generation bus, where the individual values of generation power from the two methods are not the same, even that the generation cost is the same, especially for P_{g4} , where the value of the power is kept on the upper limit, while it is never reached the upper limits at all using the proposed method, the red lines in the figure show the lower and upper limits of the generation power.

In Fig.4, Fig.5 and Fig.6, the demand power is also increased further , where, these values are chosen such that, for all of these values at least one of the generation power exceeds its maximum limit, the minimum cost of the generation power is still the same using the two methods, as it is shown in Fig.4 and the power losses from the proposed equation and the lagrange method are still close, where the proposed equation actually has less power losses valuefor certain power demand as it shown in Fig.5, from Fig.6, the individual values of the generation power of the two methods are close of the generation power of P_{g1} - P_{g4}, where theses generation units are either close to the upper limits or they are reach them, but thay are not the same for P_{g5} and P_{g6}.

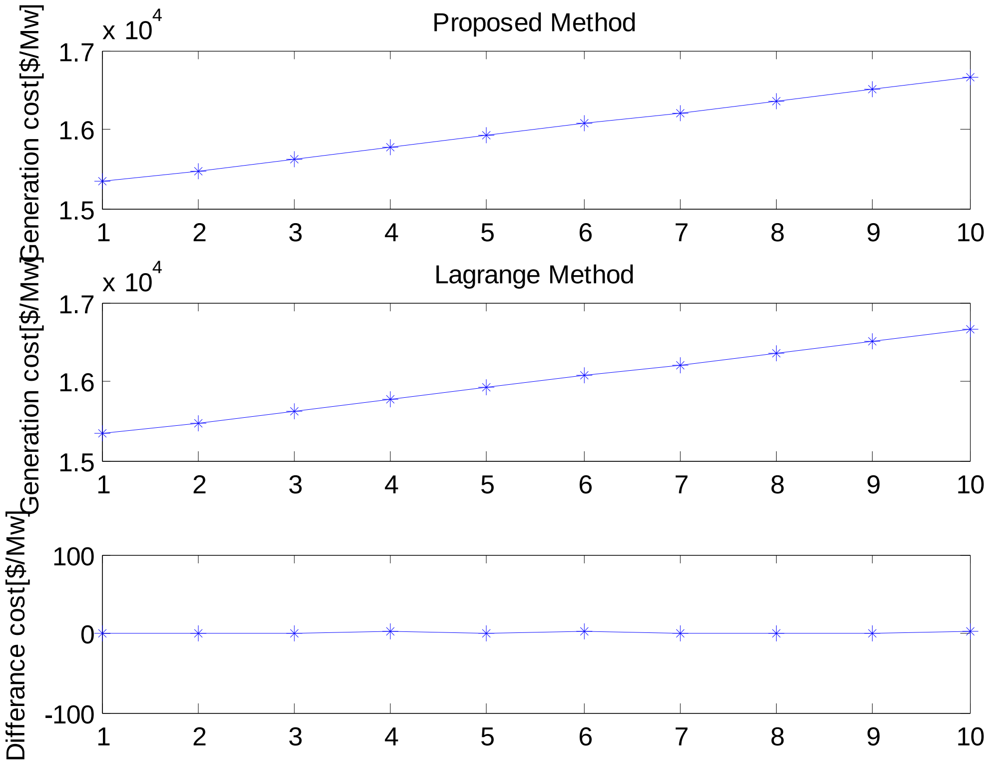

Fig.1. Total generation cost using Lagrange method and the proposed method and the difference between them.

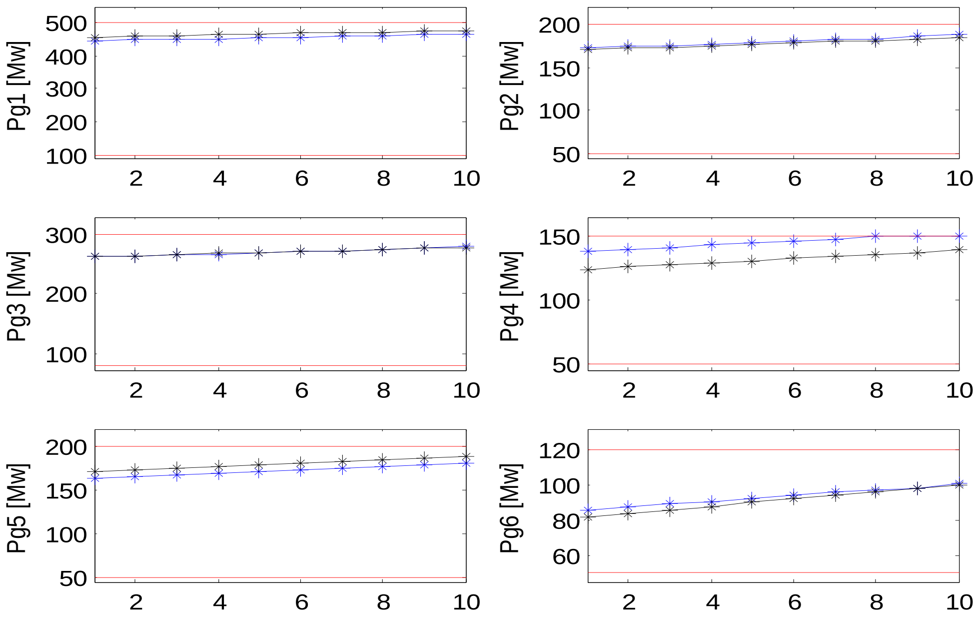

Fig.2. Generation power at each generation bus, (blue) using lagrange method and (black) using the proposed method.

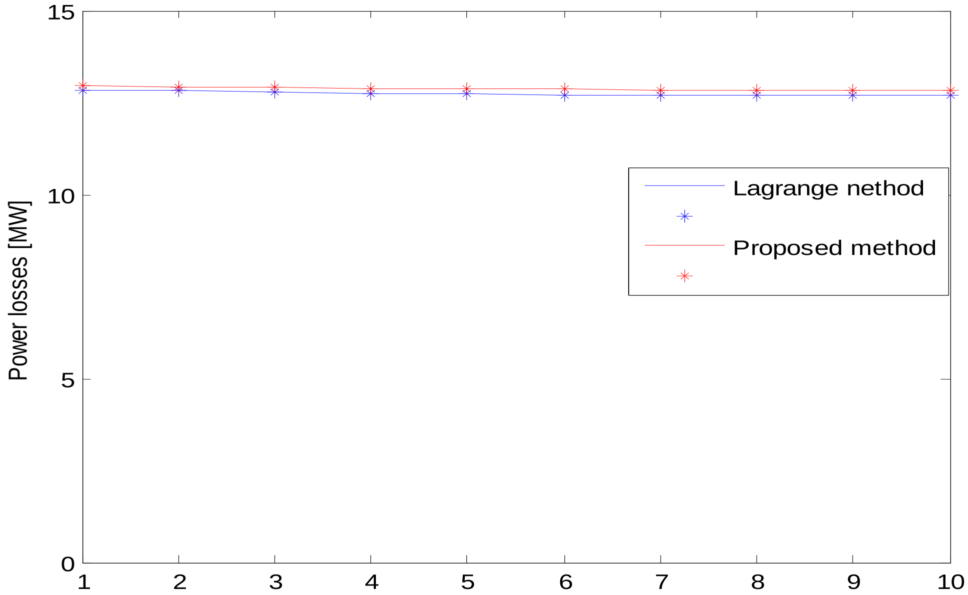

Fig. 3. Power losses using lagrange method and the proposed method.

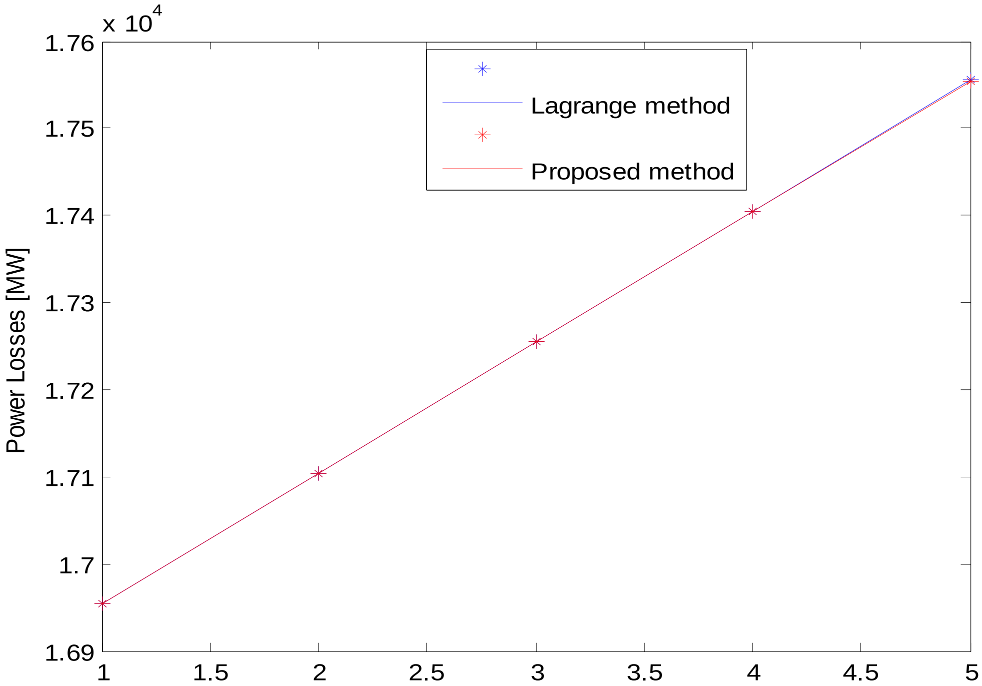

Fig.4. Total generation cost using Lagrange method and the proposed method.

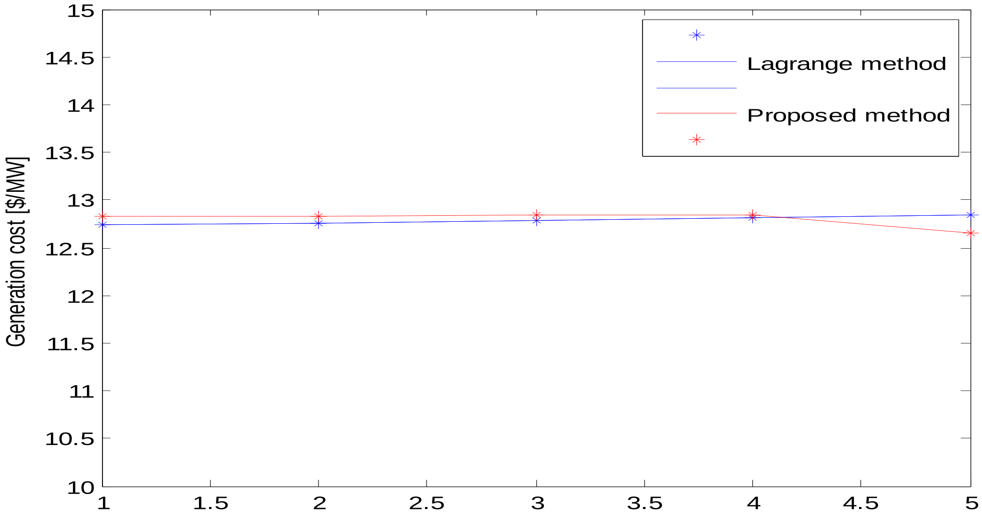

Fig.5. Power losses using lagrange method and the proposed method.

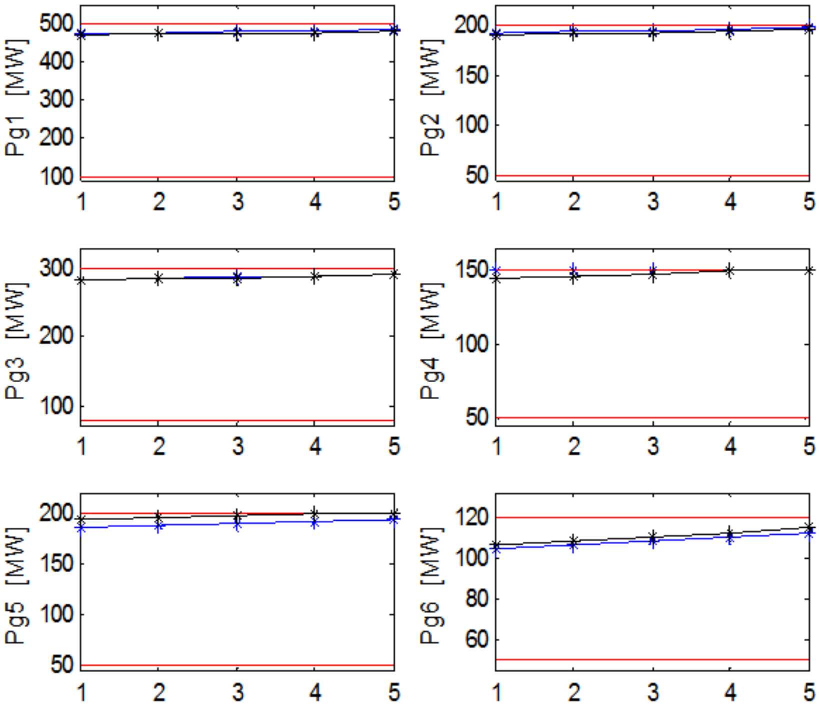

Fig.6. Generation power at each generation bus, (blue) using lagrange method and (black) using the proposed method.

V. Conclusion

The results show that, the proposed equation gives an optimal solution of IEEE 26 bus system, without need an iterative techniques, where it gave the same value of the minimum generation cost of that obtained from the Lagrange method, but the individual values of the generation powers from each generation plant are not the same, and it shows its validity even when the generation limits are violated, since the proposed equation gives the solution of OPf directly, this will reduce the computing time significantly.

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